

The impact of massive neutrinos on nonlinear matter power spectrum

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We present the first attempt to analytically study the nonlinear matter power spectrum for a mixed dark matter (cold dark matter plus neutrinos of total mass $\sim 0.1\text{eV}$) model based on cosmological perturbation theory. The suppression in the power spectrum amplitudes due to massive neutrinos is, compared to the linear regime, enhanced in the weakly nonlinear regime where standard linear theory ceases to be accurate. We demonstrate that, thanks to this enhanced effect and the gain in the range of wavenumbers to which the PT prediction is applicable, the use of such a nonlinear model may enable a precision of $\sigma(m_{\nu,\text{tot}}) \sim 0.07\text{eV}$ in constraining the total neutrino mass for the planned galaxy redshift survey, a factor of 2 improvement compared to the linear regime.

Introduction: The relic neutrinos having finite masses cause a characteristic suppression in the growth of structure formation on scales below the neutrino free-streaming scale [1]. Exploring this suppression signature from large-scale structure probes can be a vital way to constrain the neutrino masses [2]. In fact the stringent constraints, $m_{\nu,\text{tot}} \lesssim 0.2\text{--}0.6\text{eV}$, have been derived from the galaxy power spectrum and the Lyman- α forest power spectrum [3, 4, 5]. Planned galaxy surveys such as the Wide-Field Multi-Object Spectrograph (WFMOs) survey [6] will further allow a high-precision measurement of the power spectra and therefore continue to improve the sensitivity to neutrino masses (e.g., [7]).

However, most of the previous work on the subject has been based on linear perturbation theory for a mixed dark matter (MDM) model (see [8] for a review). Even at scales as large as $\sim 100\text{Mpc}$ relevant for the neutrino free-streaming scale, recent studies based on numerical techniques or perturbation theory have shown that the impact of nonlinear clustering cannot be ignored for high-precision future surveys, while these studies focused mainly on the nonlinear effect on the baryon acoustic oscillations (BAOs) in the power spectrum [9, 10]. Yet, the effects of massive neutrinos are ignored in these studies, even though the neutrinos with total mass $\gtrsim 0.06\text{eV}$, implied from the oscillation experiments, cause a $\gtrsim 5\%$ suppression in the power spectrum amplitude that surpasses the expected measurement accuracy ($\sim 1\%$) at each wavenumber band. Also unclear is how the neutrino suppression degrades the ability of BAO experiments for constraining the nature of dark energy as the neutrino effect appears at very similar scales to BAOs.

In this *Letter*, we develop a new approach to analytically study the nonlinear power spectrum for a MDM model, based on perturbation theory (PT). PT is a natural extension of the successful linear theory, and is therefore expected to give fairly accurate model predictions up

to the weakly nonlinear regime. We will then use the PT approach to study the impact of massive neutrinos on the nonlinear power spectrum, and discuss how the use of the PT prediction may help constrain the neutrino masses for WFMOs-like surveys.

Methodology: To develop a PT approach for a MDM model, we have to deal with the perturbations of multi-fluid components, cold dark matter (CDM), baryon and massive neutrinos, which are coupled to each other via gravity at redshifts of interest. Hence the expansion parameter of PT is not a single quantity in contrast to the case of a CDM dominated model in which the expansion parameter is only the amplitude of the CDM perturbations. The density perturbation field of total matter is defined as $\delta_{\text{m}} \equiv (\delta\rho_{\text{c}} + \delta\rho_{\text{b}} + \delta\rho_{\nu})/\bar{\rho}_{\text{m}} = f_{\text{cb}}\delta_{\text{cb}} + f_{\nu}\delta_{\nu}$, where the subscripts ‘m’, ‘c’, ‘b’, ‘ ν ’ and ‘cb’ stand for total matter, CDM, baryon, massive neutrinos, and CDM plus baryon, respectively, and the coefficients, f_{cb} and f_{ν} , are the fractional contributions to the matter density, $\Omega_{\text{m}0}$: $f_{\nu} \equiv \Omega_{\nu 0}/\Omega_{\text{m}0} = m_{\nu,\text{tot}}/(94.1\Omega_{\text{m}0}h^2 \text{eV})$ and $f_{\text{cb}} = 1 - f_{\nu}$. The total matter power spectrum, $P_{\text{m}}(k)$, is then given by

$$P_{\text{m}}(k) = f_{\text{cb}}^2 P_{\text{cb}}(k) + 2f_{\text{cb}}f_{\nu}P_{\text{cb},\nu}^L(k) + f_{\nu}^2 P_{\nu}^L(k), \quad (1)$$

where the power spectra with the superscript ‘L’ denote the linear-order spectra (see below) and $P_{\text{cb},\nu}^L$ is the cross spectrum between δ_{cb} and δ_{ν} .

Mixture of the neutrinos in total matter affects the nonlinear power spectrum as follows. The neutrinos would tend to remain in the linear regime rather than going into the nonlinear stage together with CDM and baryon, due to the large free-streaming. In addition, the prefactor f_{ν} appearing in Eq. (1) is likely to be small for a realistic model (e.g. $f_{\nu} \lesssim 0.07$ in [4]), allowing the nonlinear neutrino perturbations to be approximately ignored. In the following we will thus include only the linear-order neutrino perturbations, which can be accurately com-

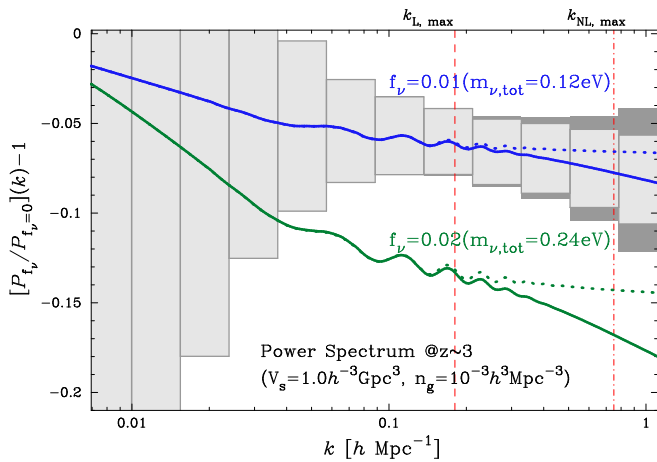


FIG. 1: Fractional difference between the mass power spectra at $z = 3$ with and without the massive neutrino contributions, where the two cases $f_\nu = 0.01$ and 0.02 are considered. The solid and dotted curves show the PT and linear theory results, respectively. The two vertical lines indicate a maximum wavenumber limit k_{\max} up to which the two models are expected to be valid (see text). The shaded boxes show the expected $1\text{-}\sigma$ errors on the power spectrum measurement for the $z \sim 3$ WFMOS survey and the case of $f_\nu = 0.01$.

puted by solving the linearized Boltzmann equations [11]. The validity of our assumption will be shown in [12]¹.

Following the standard PT approach [13], the CDM plus baryon component can be treated as a pressure-less and irrotational fluid for the scales of interest. The fluid equations for mass and momentum conservation and the Poisson equation fully describe the dynamics of the density perturbation field, δ_{cb} , and the velocity divergence field, $\theta_{\text{cb}} \equiv \nabla \cdot \mathbf{v}_{\text{cb}}/(aH)$. The solutions to this system can be obtained by making a perturbative expansion, $\delta_{\text{cb}} = \delta_{\text{cb}}^{(1)} + \delta_{\text{cb}}^{(2)} + \delta_{\text{cb}}^{(3)} + \dots$ and $\theta_{\text{cb}} = \theta_{\text{cb}}^{(1)} + \theta_{\text{cb}}^{(2)} + \theta_{\text{cb}}^{(3)} + \dots$, where the superscript ‘ i ’ denotes the i -th order perturbation. In our setting, the nonlinear correction to the total matter power spectrum $P_{\text{m}}(k)$ arises only through $P_{\text{cb}}(k)$ in Eq. (1). The nonlinear P_{cb} including the next-to-leading order corrections is expressed as

$$P_{\text{cb}}(k; z) = P_{\text{cb}}^L + P_{\text{cb}}^{(13)} + P_{\text{cb}}^{(22)}, \quad (2)$$

where the last two terms describe the nonlinear corrections, the so-called one-loop corrections, that include contributions up to the third-order perturbations.

¹ In brief we have approximately estimated the nonlinear neutrino perturbations by solving the *modified* Boltzmann equations into which the *nonlinear* gravitational potential including the contribution of the nonlinear δ_{cb} given by Eq. (3) is inserted, motivated by the fact that the nonlinear gravitational clustering is mainly driven by the CDM plus baryon perturbations. As a result, the neutrino density perturbation is found to be enhanced only by up to $\sim 10\%$ for $f_\nu \lesssim 0.05$ at scales of interest, corresponding to less than 0.01% error in the nonlinear power spectrum amplitudes due to the additional small prefactor f_ν in Eq. (1).

The neutrinos affect the spectrum P_{cb} through the effect on the linear growth rate [14]. At wavenumbers smaller than the neutrino free-streaming scale, $k_{\text{fs}}(z) \simeq 0.023(m_\nu/0.1\text{eV})[2/(1+z)]^{1/2}(\Omega_{\text{m}0}/0.23)^{1/2} h\text{Mpc}^{-1}$, the neutrinos can cluster together with CDM and baryon. Conversely, at $k > k_{\text{fs}}$, the growth rate of CDM perturbations is suppressed due to the weaker gravitational force caused by the lack of neutrino perturbations. Thus the growth rate, $D_{\text{cb}}(z, k)$, has a characteristic scale-dependence in a MDM model. This fact causes one complication in computing the second- and third-order solutions for δ_{cb} and θ_{cb} . The k -dependence of D_{cb} causes mode-couplings between the perturbations of different wavenumbers in the nonlinear regime in addition to the mode-couplings via the transfer function. Interestingly, however, we have found that, using the analytic fitting formula for D_{cb} in [14], this additional mode-coupling can be safely ignored for the expected small value of f_ν [12]. As a result, the nonlinear spectra, $P_{\text{cb}}^{(22)}$ and $P_{\text{cb}}^{(13)}$, are written in the form similar to that for a CDM model case [13]:

$$\begin{aligned} P_{\text{cb}}^{(22)}(k; z) &= \frac{k^3}{98(2\pi)^2} \int_0^\infty dr P_{\text{cb}}^L(kr; z) \\ &\times \int_{-1}^1 d\mu P_{\text{cb}}^L(k\sqrt{1+r^2-2r\mu}; z) \frac{(3r+7\mu-10r\mu^2)^2}{(1+r^2-2r\mu)^2}, \\ P_{\text{cb}}^{(13)}(k; z) &= \frac{k^3 P_{\text{cb}}^L(kr; z)}{252(2\pi)^2} \int_0^\infty dr P_{\text{cb}}^L(kr; z) \left[\frac{12}{r^2} - 158 \right. \\ &\left. + 100r^2 - 42r^4 + \frac{3}{r^2}(r^2-1)^3(7r^2+2) \ln \left| \frac{1+r}{1-r} \right| \right]. \quad (3) \end{aligned}$$

Note that $P_{\text{cb}}^{(22)}$ and $P_{\text{cb}}^{(13)}$ are roughly proportional to the square of P_{cb}^L , which enhances the neutrino effect in the nonlinear regime, compared to the linear case, P_{cb}^L .

Results: Eqs. (1) and (3) show that the PT prediction for $P_{\text{m}}(k)$ at a given redshift can be computed once the linear spectra, P_{cb}^L , $P_{\text{cb},\nu}^L$ and P_ν^L , are specified. We use the CAMB code [15] to compute the input linear spectra for a given MDM model². Fig. 1 shows the fractional difference between the power spectra at redshift $z = 3$ with and without massive neutrino contributions, where the two cases $f_\nu = 0.01$ and 0.02 are considered and other parameters are fixed to their fiducial values. This plot manifests several interesting points. First, the massive neutrinos induce a characteristic k -dependent suppression in the spectrum amplitude. For the case of linear theory, the suppression becomes nearly independent of k at very

² Our fiducial cosmological parameters are $\Omega_{\text{m}0} = 0.27$ (assuming a flat universe), $\Omega_{\text{m}0}h^2 = 0.1277$, $\Omega_{\text{b}0}h^2 = 0.0223$, $n_s = 1$, $\alpha_s = 0$, $\Delta_{\mathcal{R}}^2 = 2.35 \times 10^{-9}$, and $w = -1$, where n_s , α_s and $\Delta_{\mathcal{R}}^2$ are the primordial power spectrum parameters (tilt, running and the normalization parameter) and w is the dark energy equation of state.

small scales, $k \gg k_{\text{fs}}$, as roughly given by $\Delta P/P \sim -8f_\nu$ [2]. In contrast the PT result demonstrates that the neutrino suppression is *enhanced* in the nonlinear regime, yielding a new k -dependence in the spectrum shape.

Second, comparing the linear theory and PT results explicitly tells us the limitation of the linear theory: the linear theory is no longer accurate at $k \gtrsim 0.2h\text{Mpc}^{-1}$. More precisely, the linear theory result starts to deviate from the PT result at $k \gtrsim k_{\text{L,max}} = 0.18h\text{Mpc}^{-1}$ by $\gtrsim 1\%$ in the amplitude, as denoted by the vertical dotted line³. However PT also breaks down at scales greater than a certain maximum wavenumber limit, $k_{\text{NL,max}}$, due to a stronger mode-coupling arising from the higher-order perturbations ignored here. Using N -body simulations for a CDM model, [9] showed that the one-loop PT well matches the simulation results up to $k_{\text{NL,max}}$ given by $\Delta^2(k_{\text{NL,max}}, z) \equiv k^3 P_m(k, z)/2\pi^2|_{k=k_{\text{NL,max}}} \simeq 0.4$. The vertical dot-dashed line denotes $k_{\text{NL,max}}$ derived simply assuming this criterion for a MDM model. Thus, in the case of $z \sim 3$, the PT model may allow a factor 4 gain in k_{max} ; observationally, this is roughly equivalent to a factor $64 (= 4^3)$ gain in independent Fourier modes of the density perturbations probed for a fixed survey volume, which in turn improves the precision of the power spectrum measurement.

Can a future survey be precise enough to measure the neutrino effect? This question is partly answered in Fig. 1. The light-gray shaded boxes around the solid curve show the $1\text{-}\sigma$ measurement errors on $P(k)$ at each k bin, expected for the $z \sim 3$ WFMOS survey (see below). The neutrino suppression appears to be greater than the errors at $k \gtrsim 0.03h\text{Mpc}^{-1}$. Another intriguing consequence of the nonlinear clustering is that the amplified power of $P_m(k)$ reduces the relative importance of the shot noise contamination to the measurement errors. This can be seen by the dark-gray shaded boxes showing the $1\text{-}\sigma$ errors for the linear spectrum.

Finally it would be worth noting that wiggles in the curves reflect shifts in the BAO peak locations caused by the scale-dependent suppression effect due to neutrinos. The amount of the modulations is smaller than the measurement errors. Hence the uncertainty in neutrino mass is unlikely to largely degrade the power of BAO experiments, at least for an expected small f_ν [12].

Parameter forecasts: To realize the genuine power of future surveys for constraining the neutrino masses, we have to carefully take into account parameter degeneracies [7]. Here we estimate accuracies of the neutrino mass determination using the Fisher matrix formalism.

³ In Fig. 1, the deviation of dashed curve (linear) and solid curve (PT) around $k_{\text{L,max}}$ looks seemingly small due to the fact that, for the PT result, the denominator $P_{f_\nu=0}$ in $P_{f_\nu}/P_{f_\nu=0}$ is also computed from PT.

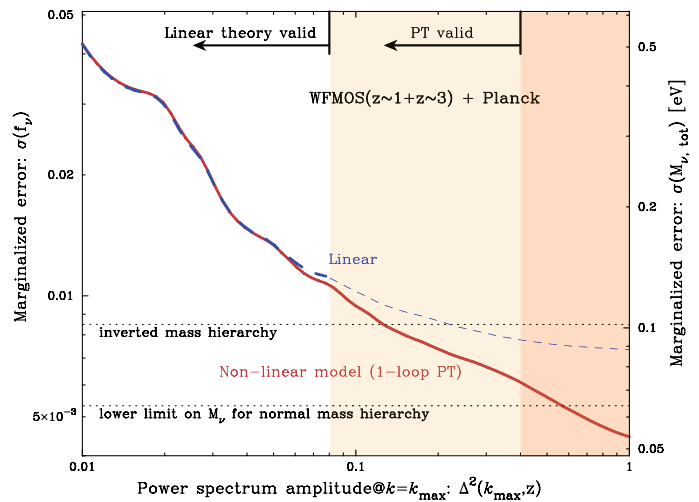


FIG. 2: The marginalized $1\text{-}\sigma$ error on the total neutrino mass as a function of the maximum wavenumber k_{max} used in each redshift slice (see text), for the WFMOS survey combined with the minimal CMB constraints. The fiducial value of $f_\nu = 0.01$ is assumed. The solid and dashed curves show the results for the PT and linear theory models, respectively. The light and dark shaded regions represent the range of k where the linear theory and the one-loop PT likely break down due to the stronger nonlinearities.

The observable we consider is the two-dimensional galaxy power spectrum given as a function of k_{\parallel} and k_{\perp} , the wavenumbers parallel and perpendicular to the line-of-sight direction [16]:

$$P_s(k_{\text{fid}\parallel}, k_{\text{fid}\perp}) = \frac{D_A(z)_{\text{fid}}^2 H(z)}{D_A(z)^2 H(z)_{\text{fid}}} [1 + \beta\mu^2]^2 b_1^2 P_m(k, z) \quad (4)$$

where $k = (k_{\perp}^2 + k_{\parallel}^2)^{1/2}$ and $\mu = k_{\parallel}/k$. Here, $k_{\perp} = [D_A(z)_{\text{fid}}/D_A(z)]k_{\text{fid}\perp}$ and $k_{\parallel} = [H(z)/H(z)_{\text{fid}}]k_{\text{fid}\parallel}$, where $D_A(z)$ and $H(z)$ are the comoving angular diameter distance and Hubble parameter, respectively. The quantities with the subscript ‘fid’ denote the quantities estimated assuming a fiducial cosmological model, which generally differs from the underlying true model. Although the equation above simply assumes the linear galaxy bias b_1 and the linear redshift distortion β , we will instead treat b_1 and β as free parameters in order not to derive too optimistic forecasts. This treatment would be adequate for our current purpose, which is to estimate how PT allows an improvement in the parameter constraints mainly due to the gain in k_{max} . A more careful analysis will be presented in detail in [12].

Following [16], the Fisher matrix for the galaxy power spectrum measurement is computed as $F_{\alpha\beta} = \int_{-1}^1 d\mu \int_{k_{\text{min}}}^{k_{\text{max}}} 2\pi k^2 dk (\partial P_s / \partial p_{\alpha}) \text{Cov}^{-1} (\partial P_s / \partial p_{\beta})$, where p_{α} represents a set of parameters and Cov^{-1} is the inverse of the covariance matrix that depends on the power spectrum itself and on survey parameters, the comoving survey volume and the number density

of galaxies. To compute $F_{\alpha\beta}$ we need to specify the integration range k_{\min} and k_{\max} ; we will throughout employ $k_{\min} = 10^{-4}h\text{Mpc}^{-1}$ to obtain the fully-convergent results, and below discuss for the choice of k_{\max} . Note that, for several redshift slices, we simply add the Fisher matrices of each slice to obtain the total Fisher matrix. The 1- σ error on a certain parameter p_α marginalized over other parameters is given by $\sigma^2(p_\alpha) = (\mathbf{F}^{-1})_{\alpha\alpha}$, where \mathbf{F}^{-1} is the inverse of Fisher matrix. We employ the WFMOS survey parameters in [16] consisting of two types of redshift surveys: the $z \sim 1$ survey covering $0.5 \leq z \leq 1.3$ with 2000 deg^2 and the $z \sim 3$ survey covering $2.5 \leq z \leq 3.5$ with 300 deg^2 . We consider 5 redshift slices. The choice of free parameters is also important for the Fisher matrix formalism: we include a fairly broad range of the model parameters given by $p_\alpha = \{\Omega_{m0}, \Omega_{m0}h^2, \Omega_{b0}h^2, f_\nu, n_s, \alpha_s, \Delta_{\mathcal{R}}^2, w, \beta(z_i), b_1(z_i)\}$. We assume three neutrino species that are totally mass degenerate and adopt $f_\nu = 0.01$ as the fiducial value. The fiducial $\beta(z_i)$ and $b_1(z_i)$ for the i -th redshift slice are computed following [16]. In total we include 18 free parameters.

Fig. 2 demonstrates the marginalized 1- σ errors on the total neutrino mass as a function of k_{\max} , where the galaxy power spectrum over a range of $k_{\min} \leq k \leq k_{\max}$ is included. The value of k_{\max} for each redshift slice is specified by inverting $\Delta^2(k_{\max}; z_i)$ for the value given in the horizontal axis. The errors shown here are for the WFMOS survey combined with the CMB information on cosmological parameters except for the neutrino masses, f_ν , and the dark energy parameter, w . The solid and dashed curves show the results for the PT and linear theory, respectively. If the linear theory is employed, a reliable accuracy to be obtained is $\sigma(m_{\nu,\text{tot}}) \simeq 0.13\text{eV}$ in order not to have a biased constraint due to the inaccurate model prediction. On the other hand, if the PT prediction is valid up to $\Delta^2(k_{\max}) \simeq 0.4$ as discussed in Fig. 1, the accuracy of $\sigma(m_{\nu,\text{tot}}) \simeq 0.072\text{eV}$ may be attainable, a factor of 2 improvement.

It should be also noted that a wide redshift coverage for the planned WFMOS survey is very efficient to break parameter degeneracies, especially between the neutrino mass and the dark energy parameters [17, 18], because the dark energy is likely to affect gravitational clustering only at low redshifts, $z \lesssim 1$.

Discussion: It is of great importance to carefully study nonlinear structure formation for a most realistic model, i.e. a MDM model including $\sim 0.1\text{eV}$ neutrinos, in preparation for future galaxy surveys. While the PT model developed in this *Letter* gives the first step in this direction, another complement to the analytic approach is to use a hybrid N -body simulation consisting of cold and hot particles, which seems feasible with current numerical resources by extending the pioneering work [19]

for a model of $\sim 10\text{eV}$ neutrinos to models of $\sim 0.1\text{eV}^4$. PT will also play a useful role in calibrating/checking the simulations results.

We have demonstrated that the use of PT may enable an improvement in the neutrino mass constraint by a factor 2 compared to the case that linear theory is used, for the planned WFMOS survey. However our study involves several idealizations: most importantly we assumed the linear galaxy bias and the linear redshift distortion. At least for the large scales $\sim 100\text{Mpc}$, it seems feasible to develop a self-consistent model to describe galaxy clustering observables including the non-linear effects on the galaxy bias and redshift distortions for a MDM model, by using the perturbation theory approach [21] and/or the halo model approach and by combining with simulations. Such a refined model to describe galaxy clustering observables in the weakly nonlinear regime would be worth exploring in order to exploit the full potential of the forthcoming galaxy surveys for constraining or even *determining* the neutrino masses.

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⁴ After we submitted our paper, [20] presented a simulation based study for a MDM model where the similar conclusion, the enhanced neutrino effect in the nonlinear regime, was found.

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